

# Concept-Development Practice Page

13-3

## Gravitational Interactions

The equation for the law of universal gravitation is

$$F = G \frac{m_1 m_2}{d^2}$$

where  $F$  is the attractive force between masses  $m_1$  and  $m_2$  separated by distance  $d$ .  $G$  is the universal gravitational constant (and relates  $G$  to the masses and distance as the constant  $\pi$  similarly relates the circumference of a circle to its diameter). By substituting changes in any of the variables into this equation, we can predict how the others change. For example, we can see how the force changes if we know how either or both of the masses change, or how the distance between their centers changes.

Suppose, for example, that one of the masses somehow is doubled. Then substituting  $2m_1$  for  $m_1$  in the equation gives

$$F_{\text{new}} = G \frac{2m_1 m_2}{d^2} = 2G \frac{m_1 m_2}{d^2} = 2F_{\text{old}}$$

So we see the force doubles also. Or suppose instead that the distance of separation is doubled. Then substituting  $2d$  for  $d$  in the equation gives

$$F_{\text{new}} = G \frac{m_1 m_2}{(2d)^2} = G \frac{m_1 m_2}{4d^2} = \frac{1}{4} G \frac{m_1 m_2}{d^2} = \frac{1}{4} F_{\text{old}}$$

And we see the force is only  $1/4$  as much.

Use this method to solve the following problems. Write the equation and make the appropriate substitutions.

1. If both masses are doubled, what happens to the force?

$$F = G \frac{m_1 m_2}{d^2}$$

$$F = G \frac{2m_1 2m_2}{d^2} \rightarrow 2 \times 2 = \textcircled{4x} \text{ The force is 4 times its original amount}$$

2. If the masses are not changed, but the distance of separation is reduced to half the original distance, what happens to the force?

$$F = G \frac{m_1 m_2}{d^2}$$

$$F = G \frac{m_1 m_2}{\frac{1}{2}d^2} \rightarrow \frac{1}{\frac{1}{2}^2} = \frac{1}{\frac{1}{4}} = \textcircled{4x} \text{ The force is 4 times its original amount.}$$

GRAVITY  
SIGH



3. If the masses are not changed, but the distance of separation is reduced to one fourth the original distance, what happens to the force?

~~$$F = G \frac{m_1 m_2}{d^2}$$~~

$$F = G \frac{m_1 m_2}{\frac{1}{4}d^2} \rightarrow \frac{1}{\frac{1}{4}d^2} = \frac{1}{\frac{1}{16}} = 16 \times$$

The new force is 16 times the original force

EQUATIONS ARE "GUIDES TO THINKING"

4. If both masses are doubled, and the distance of separation is doubled, show what happens to the force.

$$F = G \frac{m_1 m_2}{d^2}$$

$$F = G \frac{2m_1 2m_2}{2d^2} \rightarrow \frac{2 \times 2}{2^2} = \frac{4}{4} = 1 \times$$

The new force stays the same.



5. If one of the masses is doubled, the other remains unchanged, and the distance of separation is tripled, show what happens to the force.

$$F = G \frac{m_1 m_2}{d^2}$$

$$F = G \frac{2m_1 m_2}{3d^2} \rightarrow \frac{2 \times 1}{3^2} = \frac{2}{9} \times$$

The new force is  $\frac{2}{9}$  times the original force

6. Consider a pair of binary stars that pull on each other with a certain force. Would the force be larger or smaller if the mass of each star were three times as great when their distance apart is three times as far? Show what the new force will be compared to the first one.

$$F = G \frac{m_1 m_2}{d^2}$$

$$F = G \frac{3m_1 3m_2}{3d^2} = \frac{3 \times 3}{3^2} = \frac{9}{9} = 1 \times$$

The force stays the same.

## CONCEPTUAL PHYSICS





**PRACTICE**

Solutions on the next two pages

Answer the following problems. Write your answers using scientific notation.

1. Calculate the force between two objects that have masses of 70 kilograms and 2,000 kilograms separated by a distance of 1 meter.
2. Calculate the force between two touching grapefruits each with a radius of 0.08 meters and a mass of 0.45 kilograms.
3. Calculate the force between one grapefruit as described above and Earth. Earth has a mass of  $5.9742 \times 10^{24}$  kg and a radius of  $6.3710 \times 10^6$  meters. Assume the grapefruit is resting on Earth's surface.
4. A man on the moon with a mass of 90 kilograms weighs 146 newtons. The radius of the moon is  $1.74 \times 10^6$  meters. Find the mass of the moon.
- ~~5. For  $m = 5.9742 \times 10^{24}$  kilograms and  $r = 6.378 \times 10^6$  meters, what is the value given by this equation:  $G \frac{m}{d^2}$ ?~~
  - ~~a. Write down your answer and simplify the units.~~
  - ~~b. What does this number remind you of?~~
  - ~~c. What real-life values do  $m$  and  $r$  correspond to?~~
6. The distance between Earth and its moon is  $3.84 \times 10^8$  meters. Earth's mass is  $m = 5.9742 \times 10^{24}$  kilograms and the mass of the moon is  $7.36 \times 10^{22}$  kilograms. What is the force between Earth and the moon?
7. A satellite is orbiting Earth at a distance of 35 kilometers. The satellite has a mass of 500 kilograms. What is the force between the planet and the satellite?
8. The mass of the sun is  $1.99 \times 10^{30}$  kilograms and its distance from Earth is 150 million kilometers ( $150 \times 10^9$  meters). What is the gravitational force between the sun and Earth?

Solutions on the next  
two pages



## Universal Gravitation Practice

1)  $m_1 = 70 \text{ kg}$   
 $m_2 = 2,000 \text{ kg}$

$d = 1 \text{ m}$

$G = 6.67 \times 10^{-11}$

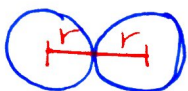
$F = ?$

$$F = G \frac{m_1 m_2}{d^2}$$

$$F = (6.67 \times 10^{-11}) \left( \frac{70 \times 2,000}{1^2} \right)$$

$$\boxed{F = 9.34 \times 10^{-6} \text{ N or } 0.00000934 \text{ N}}$$

2)



$r + r = d$

$m_1 = 0.45 \text{ kg}$

$m_2 = 0.45 \text{ kg}$

$d = 0.08 + 0.08 = 0.16 \text{ m}$

$G = 6.67 \times 10^{-11}$

$F = ?$

$$F = G \frac{m_1 m_2}{d^2}$$

$$F = 6.67 \times 10^{-11} \left( \frac{0.45 \times 0.45}{(0.16)^2} \right)$$

$$\boxed{F = 5.28 \times 10^{-14} \text{ N}}$$

3)  $m_1 = 5.97 \times 10^{24} \text{ kg}$

$m_2 = 0.45 \text{ kg}$

$d = 6.37 \times 10^6 + 0.08$

$G = 6.67 \times 10^{-11}$

$F = ?$

$$F = G \frac{m_1 m_2}{d^2}$$

$$F = 6.67 \times 10^{-11} \left( \frac{(5.97 \times 10^{24})(0.45)}{(6.37 \times 10^6 + 0.08)^2} \right)$$

$$\boxed{F = 4.42 \text{ N}}$$

4)  $m_1 = 90 \text{ kg}$

$m_2 = ?$

$F = 146 \text{ N}$

$d = 1.74 \times 10^6 \text{ m}$

$G = 6.67 \times 10^{-11}$

$$d^2 \times F = G \frac{m_1 m_2}{d^2} \times d^2$$

$$\frac{F d^2}{G m_1} = \frac{G m_1 m_2}{G m_1}$$

$$\frac{F d^2}{G m_1} = m_2$$

$$\frac{(146)(1.74 \times 10^6)^2}{(6.67 \times 10^{-11})(90)} = m_2$$

$$\boxed{7.36 \times 10^{22} \text{ kg} = m_2}$$

6)  $d = 3.84 \times 10^8 \text{ m}$   
 $m_1 = 5.97 \times 10^{24} \text{ kg}$   
 $m_2 = 7.36 \times 10^{22} \text{ kg}$   
 $G = 6.67 \times 10^{-11}$   
 $F = ?$

$$F = G \frac{m_1 m_2}{d^2}$$

$$F = 6.67 \times 10^{-11} \left( \frac{(5.97 \times 10^{24})(7.36 \times 10^{22})}{(3.84 \times 10^8)^2} \right)$$

$$F = 2 \times 10^{20} \text{ N}$$

7)  $d = 35 \text{ km} \rightarrow 35000 \text{ m}$   
 $m_1 = 500 \text{ kg}$   
 $m_2 = 5.97 \times 10^{24} \text{ kg}$   
 (mass of earth taken from previous problem)  
 $G = 6.67 \times 10^{-11}$   
 $F = ?$

$$F = G \frac{m_1 m_2}{d^2}$$

$$F = 6.67 \times 10^{-11} \left( \frac{(500 \text{ kg})(5.97 \times 10^{24})}{(35000 \text{ m})^2} \right)$$

$$F = 1.63 \times 10^8 \text{ N}$$

8)  $m_1 = 1.99 \times 10^{30} \text{ kg}$   
 $m_2 = 5.97 \times 10^{24} \text{ kg}$   
 (mass of earth taken from previous problem)  
 $d = 150 \times 10^9 \text{ m}$   
 $G = 6.67 \times 10^{-11}$   
 $F = ?$

$$F = G \frac{m_1 m_2}{d^2}$$

$$F = 6.67 \times 10^{-11} \left( \frac{(1.99 \times 10^{30})(5.97 \times 10^{24})}{(150 \times 10^9)^2} \right)$$

$$F = 3.52 \times 10^{22} \text{ N}$$